Positron energy calibration.

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This note discusses the systematic uncertainty caused by an error in the positron energy scale. Sensitivities of all the four Michel parameters to the energy scale are obtained from fits to generated 2-d spectra. A method of determining the energy scale is described. Current status of its implementation is given. Preliminary data on performance of the energy calibration method are presented.

Introduction

The positron energy scale defines a distortion of the 2-dimensional Michel spectrum in the form $E \rightarrow (1 + \beta)E$ where $E$ is the positron energy. An error in a value of magnetic field used by the reconstruction program can introduce this kind of distortion. The goal of the energy calibration procedure is to determine $\beta$ of the reconstructed spectrum.

Technically calibrations for the upstream and downstream parts of the spectrum will be done separately. That gives us $\beta_{\text{up}}$ and $\beta_{\text{down}}$, or equivalently a global e-scale $\beta_g$ and an asymmetry $\Delta$:

$$
\beta_{\text{up}} = (1 + \beta_g) (1 + \Delta) \\
\beta_{\text{down}} = (1 + \beta_g) (1 - \Delta)
$$

Sensitivity of a Michel parameter $\alpha$ to the global energy scale is the derivative $\partial \alpha / \partial \beta_g$ showing how an error in the energy scale translates into an error in the parameter. Similarly for the asymmetry $\Delta$.

In this note instead of $E$ a dimensionless variable $x$ is used. It is the fractional energy $x = E / E_{\text{max}}$. $E_{\text{max}}$ being the upper kinematic limit on the positron energy. $E_{\text{max}} \approx 52.83 \text{MeV}$.

A Michel spectrum function which includes the first order radiative corrections was used in the study. The same function was used to generate spectra and to fit them. Muon polarization was assumed to be 100%.

All histogramming and fitting was done using the ROOT framework [1]. ROOT fitting subroutines are based on the well known MINUIT tool [2]. All simulations used a ROOT implementation of the Mersenne Twistor random number generator [3] with the period $2^{10037} - 1$. 

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1 Sensitivity of Michel parameters to the energy scale.

"Events" \{x, cos(\theta)\} were produced by sampling the Michel spectrum using the acceptance-rejection method. Standard Model values of Michel parameters were used in generation. One data set consisted of 10^9 events in the whole spectrum.

A set of 2D histograms with 300 bins in the [0.3, 0.98] dimensionless energy range and 200 bins in the [−0.98, 0.98] cos(\theta) range each was defined. To fill the histograms only events with |cos(\theta)| ≥ 0.5 were used. The angular cut is consistent with the flat acceptance region defined in [4], but the energy range used extends below the 20MeV boundary quoted in [4].

During the filling \(x\) was scaled as \((1 + \Delta) x\) for events with cos(\theta) < 0 and as \((1 - \Delta) x\) for events with cos(\theta) > 0, thus introducing the asymmetry \(\Delta\). The asymmetry varied from \(-15 \cdot 10^{-4}\) to \(+15 \cdot 10^{-4}\) with the step \((15/7) \cdot 10^{-4}\). This set of histograms was fitted with the nominal Michel spectrum function. Four Michel parameters: \(\rho, \eta, \xi, \delta\) and a global normalization varied freely during the fits. Maximum log-likelihood fits were employed.

Deviations of obtained Michel parameters from their nominal values were plotted against the energy scale asymmetry, see Fig. 1.

To obtain sensitivities of the parameters to the global energy scale the Michel spectrum function where argument \(x\) was replaced by \((1 + \beta_2) x\) was fitted to a histogram filled with \(\Delta = 0\). Like for the \(\Delta\), 15 points with \(\beta_2\) varying between \(-15 \cdot 10^{-4}\) to \(+15 \cdot 10^{-4}\) were calculated. The results can be seen on Fig. 2.

Obviously deviations of the Michel parameters are linear in \(\beta_2\) and \(\Delta\). The plots also show results of straight line fits to the data points. Slope of a line \(p_1\) is the sensitivity of the corresponding parameter to the asymmetry or the global energy scale. Note that the error in \(p_1\) on the plots is not reliable. The data points are correlated since all of them are obtained from the same statistics. To get a handle on variations of the sensitivities another statistically independent data set of 10^9 events was processed in the same way. The results are summarized in the Table 3. One can see that the result is reproducible and the errors in sensitivities are probably overestimated by the straight line fits.

2 The energy calibration procedure.

The sharp end of the positron energy spectrum at the upper kinematic limit provides a natural calibration point. However positrons lose energy traveling in the detector material. Due to this process a systematic shift in the end point of the reconstructed spectrum should be expected.

A procedure to make the energy scale calibration taking into account the energy loss is described in previous TWIST documents, see for example [5]. It's based on the fact that the mean energy loss for a positron with some fixed initial energy is

\[
\langle \Delta E \rangle = \frac{\alpha}{|\cos(\theta)|}
\]
Figure 1: Dependence of Michel parameters on the energy scale asymmetry $\Delta$.

Figure 2: Dependence of Michel parameters on the global energy scale $\beta\gamma$. 
<table>
<thead>
<tr>
<th></th>
<th>Data set 1</th>
<th>Data set 2</th>
<th>Difference between data sets</th>
<th>Error from the fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\partial p/\partial \Delta$</td>
<td>1.307</td>
<td>1.306</td>
<td>$1 \cdot 10^{-3}$</td>
<td>$59 \cdot 10^{-3}$</td>
</tr>
<tr>
<td>$\partial \eta/\partial \Delta$</td>
<td>-10.79</td>
<td>-11.02</td>
<td>0.23</td>
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<td>$\partial \xi/\partial \Delta$</td>
<td>5.436</td>
<td>5.43</td>
<td>$6 \cdot 10^{-3}$</td>
<td>$76 \cdot 10^{-3}$</td>
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<tr>
<td>$\partial \delta/\partial \Delta$</td>
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<td>-4.947</td>
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<td>$43 \cdot 10^{-3}$</td>
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<tr>
<td>$\partial p/\partial \beta_g$</td>
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<td>-1.047</td>
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<td>$59 \cdot 10^{-3}$</td>
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<td>$\partial \eta/\partial \beta_g$</td>
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<td>3.626</td>
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<td>$\partial \delta/\partial \beta_g$</td>
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<td>-0.8404</td>
<td>$0.3 \cdot 10^{-3}$</td>
<td>$43 \cdot 10^{-3}$</td>
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</tbody>
</table>

Figure 3: Variation in sensitivities from two different data sets compared to the straight line fit error.

This equation is rigorously valid for the planar detector geometry.
Figure 4: Examples of the end point fitting.

References


