Tests of $\square$-decay theory
Tests of $\Box(p_e)$

$\Box(p_e) \equiv \frac{d\Box(p_e)}{dp_e} \Box \frac{g^2}{2\hbar^7 \Box^3 c^3} |M_{fi}|^2 F(Z', p_e) S(p_e, p_{\Box}) \left(E_f \Box E_e\right)^2 p_e^2$

Plot --

$\sqrt{\frac{\Box(p_e)}{F(Z', p_e)p_e^2}} = K \left(E_f \Box E_e\right) \quad \text{Kurie plot for “allowed”}$

-- Find endpoint energy/momentum

$\sqrt{\frac{\Box(p_e)}{F(Z', p_e)S(p_e, p_{\Box})p_e^2}} = K \left(E_f \Box E_e\right) \quad \text{Kurie plot for “forbidden”}$
Tests of $\Box(p_e)$

- If a (allowed) Kurie plot is linear then --
  - The transition (decay) is "allowed" which implies $|M_{fi}|^2$ is a constant; $|M_{fi}|^2$ does not depend on $k_{\Box}$ and/or $k_{\square}$.
  - The extrapolated x-axis crossing (y=0) is at $E_e = E_f$ known as the endpoint energy.
  - The slope of the line is --

\[
\frac{g^2}{2\hbar^7\Box^3c^3} |M_{fi}|^2
\]

Weak interaction coupling constant

Nuclear matrix element

Two unknowns --- c.f., Fig. 9.4
Tests of $\Box(p_e)$

- If a (allowed) Kurie plot is not linear then --
- The transition (decay) is not “allowed” which implies $|M_{fi}|^2$ is a not constant; $|M_{fi}|^2$ does depend on $k$ and/or $k$. The decay must be “first forbidden” or “second forbidden” … (see next slide)
- A functional correction can be applied to straighten the line and thereby represent the momentum dependence, called the “shape correction” -- (see slide 2)

$S(p_e, p_{\Box})$

c.f. Fig. 9.5
Tests of $\mathcal{O}(p_e)$

Lepton wavefunctions --

$$e^{ik_e \cdot \vec{r}} \frac{1}{\sqrt{V}} + i k_e \cdot \vec{r} + \frac{(k_e \cdot \vec{r})^2}{2} + \cdots \frac{1}{\sqrt{V}}$$

"Allowed term"

"First forbidden term"

"Second forbidden term"

etc....
Tests of \( \square \)

\[
\square = \int_0^{p_{\text{max}}} \frac{d\square(p_e)}{dp_e} dp_e = \int_0^{p_{\text{max}}} \frac{d\square(p_e)}{dp_e} dp_e
\]

Total decay rate

\[
\square = \frac{g^2}{2\hbar^7 \hbar^3 c^3} \left| M_{\text{fi}} \right|^2 \int_0^{p_{\text{max}}} F(Z', p_e) \left( E_f - E_e \right)^2 p_e^2 dp_e
\]

\( f(Z', p_o) \equiv \frac{1}{m_e^5 c^7} \int_0^{p_{\text{max}}} F(Z', p_e) \left( E_f - E_e \right)^2 p_e^2 dp_e \)

\[
\square = \frac{g^2 m_e^5 c^7}{2\hbar^7 \hbar^3 c^3} \left| M_{\text{fi}} \right|^2 \int_0^{p_{\text{max}}} f(Z', p_o) \]

“Fermi integral”

Can be calculated!

c.f., Fig. 9.8
Tests of \( \Box \)

\[
\Box = \frac{g^2 m_e c^4}{2\hbar^7 \Box^3} |M_{fi}|^2 f(Z', p_o)
\]

\[
\Box = \frac{\text{Ln}2}{t_{1/2}} ; \quad \frac{\text{Ln}2}{t_{1/2}} = \frac{g^2 m_e c^4}{2\hbar^7 \Box^3} |M_{fi}|^2 f(Z', p_o)
\]

Comparative half-life

-- or --

\( ft_{1/2} \)

\[
ft_{1/2} = (\text{Ln}2) \frac{2\hbar^7 \Box^3}{g^2 m_e c^4 |M_{fi}|^2}
\]

Differences in \( ft_{1/2} \)

must be due to differences in

\[ |M_{fi}|^2 \]

\( ft \)-value

- or \((\log_{10} ft)\) value
Tests of

For what nuclear decays might \(|M_{fi}|^2\) be ~the same?

How would you know they were the same?

Log ft would be ~same

If \(|M_{fi}|^2\) can be estimated then g can be determined!!

0+ 0+ decays

c.f., Table 9.2
Neutrino mass (Why?)

\[ Q = \left( M^N_p \square M^N_D \square m_e \square m_\square \right) c^2 \]

\[ m_\square > 0 \square Q \text{ will decrease!} \quad \text{Limits on } m_\square \text{ are } \sim \text{few eV} \]

\[ Q = T_D + T_e + T_\square \]

Need to know everything to a precision/accuracy \( \square \sim \text{few eV} \)

\[ T_e = Q \square T_D \square T_\square \]

\[ T_{e \text{ max}} = Q \square T_D ; \quad T_\square = 0 \]

\[ m_\square > 0 \square T_{e \text{ max}} \text{ will decrease!} \]

\[ \square(p_e) \equiv \frac{d\square(p_e)}{dp_e} \square \frac{g^2}{2h^7 \square^3 c^3} |M_{fi}|^2 F(Z', p_e) S(p_e, p_\square) (E_f \square E_e)^2 p_e^2 \]

Need to know nuclear wavefunctions!!

\[ \begin{array}{c}
\square \quad 3\text{H} \quad \square \quad 3\text{He} \quad \square \quad \square \quad + \quad \square + \quad \square \quad \square \quad e
\end{array} \]