Parity Conservation in the weak (beta decay) interaction
The parity operation

The *parity operation* involves the transformation

In rectangular coordinates --
\[ x \uparrow \downarrow x, \quad y \uparrow \downarrow y, \quad z \uparrow \downarrow z \]

In spherical polar coordinates --
\[ r \uparrow r, \quad \phi \uparrow (\phi + \phi), \quad \theta \uparrow (\theta + \theta) \]
In quantum mechanics

For states of *definite* (unique & constant) *parity* -

\[ \hat{P} (x, y, z) = \pm 1 \hat{P} (\hat{x}, \hat{y}, \hat{z}) \]

If the parity operator *commutes with hamiltonian* -

\[ [\hat{P}, \hat{H}] = 0 \]

The parity is a “constant of the motion”

Stationary states must be states of *constant parity*

e.g., ground state of \(^2\text{H}\) is \(s \ (l=0) + \text{small} \ d \ (l=2)\)
In quantum mechanics

To *test parity conservation* -
- Devise an experiment that could be done:
  (a) In one configuration
  (b) In a parity “reflected” configuration
- If both experiments give the “same” results, parity is conserved -- it is a good symmetry.
Parity operations --

Parity operation on a *scalar* quantity -
\[ \hat{P} E = E \quad \hat{P} (\vec{r} \cdot \vec{r}) = (\vec{r} \cdot \vec{r}) \]

Parity operation on a *polar vector* quantity -
\[ \hat{P} \vec{r} = \vec{r} \quad \hat{P} \vec{p} = \vec{p} \]

Parity operation on a *axial vector* quantity -
\[ \hat{P} \vec{L} = \hat{P} (\vec{r} \vec{p}) = (\vec{r} \vec{p}) = \vec{L} \]

Parity operation on a *pseudoscalar* quantity -
\[ \hat{P} (\vec{p} \cdot \vec{L}) = \vec{p} \cdot \vec{L} \]
If parity is a good symmetry...

- The decay should be the same whether the process is parity-reflected or not.
- In the Hamiltonian, $V$ *must not* contain terms that are pseudoscaler.
- If a *pseudoscaler dependence is observed* - parity symmetry is violated in that process - *parity is therefore not conserved.*


http://publish.aps.org/
Look at the angular distribution of decay particle (e.g., red particle). If this is symmetric above/below the mid-plane, then --

\[ \langle \vec{p} \cdot \vec{I} \rangle = 0 \]
If parity is a good symmetry...

- The $\left\langle \vec{p} \cdot \vec{I} \right\rangle = 0$ the decay intensity should not depend on $\left( \vec{p} \cdot \vec{I} \right)$.

- If $\left\langle \vec{p} \cdot \vec{I} \right\rangle \neq 0$ there is a dependence on $\left( \vec{p} \cdot \vec{I} \right)$ and parity is not conserved in beta decay.
Discovery of parity non-conservation (Wu, et al.)

Consider the decay of $^{60}$Co

Conclusion: $G-T$, allowed


http://publish.aps.org/
GT: $\square I = 1$

$A: 1 - \frac{1}{3} \frac{v}{c} \cos \theta$

$H_\square = \frac{\sqrt{c}}{c}$

$H_\square = \square 1$

$H_\square = \frac{\sqrt{c}}{c}$

$H_\square = \square 1$

$H_\square = \square 1$

$F: I = 0 \square I = 0$

$V: 1 + \frac{v}{c} \cos \theta$

$H_\square = \frac{\sqrt{c}}{c}$

$H_\square = \square 1$

$H_\square = \square 1$

Not observed

Measure $T_{\text{recoil}}$
Conclusions

GT: $\Box I = 1$

$A: 1 \Box \frac{1}{3} \frac{v}{c} \cos \Box$

GT is an axial-vector

Violates parity

$F: I = 0 \Box I = 0$

$V: 1 + \frac{v}{c} \cos \Box$

$H_{\Box} = \Box \frac{v}{c}$

$H_{\Box} = 1$

F is a vector

Conserves parity
Implications

Inside the nucleus, the N-N interaction is

\[ V_N = V_s + V_w \]

Conserves parity

Can violate parity

The nuclear state functions are a superposition

\[ \psi = \psi_s + F \psi_w ; \quad F \approx 10^{-7} \]

Nuclear spectroscopy not affected by \( V_w \)
Generalized $\Box$-decay

The hamiltonian for the vector and axial-vector weak interaction is formulated in Dirac notation as --

\[
H = g \left( \gamma_p \gamma_\mu \gamma_5 n \right) \left( \gamma_e \gamma_\mu \gamma_5 \bar{n} \right) + h.c. = gV
\]

\[
H = g \left( \gamma_p \gamma_\mu \gamma_5 n \right) \left( \gamma_e \gamma_\mu \gamma_5 \bar{n} \right) + h.c. = gA
\]

Or a linear combination of these two --

\[
H \Box g \left( C_V V + C_A A \right)
\]
**Generalized \( \square \)-decay**

The *generalized hamiltonian* for the weak interaction that includes parity violation and a two-component neutrino theory is --

\[
H = g \sum_{i=A,V} C_i \left( \square_p^* O_i \square_n \right) \left[ \square_e^* O_i \left(1 + \square_5\right) \square \right] + h.c.
\]

\[
O_V = \square_1; \quad O_V = \square_5
\]

Empirically, we need to find -- \( g \) and \( C_A/C_V \)

Study: \( n \square p \) (mixed \( F \) and \( \text{GT} \)),
and: \( ^{14}O \square ^{14}N^* \) (\( I=0 \square \) \( I=0 \); pure \( F \))
Generalized □-decay

$^{14}O \square^{14}N^* \text{ (pure F)}$

$$ft = \frac{2 \hbar^7 \log 2}{m^5 c^4 |M_F|^2 (g^2 C_V^2)}$$

$$\left( g^2 C_V^2 \right) = 1.4029 \pm 0.0022 \square 10^{-49} \text{ erg cm}^3$$

$n \square p \text{ (mixed F and GT)}$

$$ft = \frac{2 \hbar^7 \log 2}{g^2 m^5 c^4 \left[ C_V^2 \right] |M_F|^2 + \left( C_A^2 \right) |M_{GT}|^2}$$
Generalized $\square$-decay

Assuming simple (reasonable) values for the square of the matrix elements, we can get (by taking the ratio of the two ft values --

$$\frac{2C_V^2}{C_V^2 + 3C_A^2} = 0.3566 \quad \frac{C_V^2}{C_A^2} = 1.53$$

Experiment shows that $C_V$ and $C_A$ have **opposite signs**.
Universal Fermi Interaction

In general, the fundamental weak interaction is of the form --

\[ H \quad g(V \square A) \]

- \( n \square p + \square \bar{p} + \square e \)  
  semi-leptonic weak decay
- \( \square \bar{e} \quad e^\pm + \square \bar{e} + \square \)  
  pure-leptonic weak decay
- \( \square \bar{\nu} \quad \nu^\pm + \square \bar{\nu} \)  
  semi-leptonic weak decay
- \( \square^o \quad \bar{\nu}^\pm + p \)  
  Pure hadronic weak decay

All follow the (V-A) weak decay.  (c.f. Feynman’s CVC)
Universal Fermi Interaction

In general, the fundamental weak interaction is of the form --

\[ H \ g(V \cdot A) \]

BUT -- is it really that way - absolutely?

How would you proceed to test it?

\[ \mu^- e^- + \bar{\nu}_e + \nu_e \quad \text{pure-leptonic weak decay} \]

The Triumf Weak Interaction Symmetry Test - TWIST
Other symmetries

Charge symmetry - $C$

\[ n \, p + \n^+ + e^- \quad C \quad \bar{n} \, \bar{p} + \bar{\n}^+ + \bar{e}^- \]

All vectors unchanged

Time symmetry - $T$

\[ n \, p + \n^+ + e^- \quad T \quad n \, p + \n^+ + e^- \]

\[ \n^- + n \, p + \n^- \quad (\text{Inverse } \downarrow \text{-decay}) \]

All time-vectors changed (opposite)
Symmetries in weak decay

\[ \vec{s}_s = 0 \]

\( \square^+ \) at rest

Note helicities of neutrinos

\[ \hat{m} - \hat{n} \]

\[ \hat{m} + \hat{n} \]
Symmetries in weak decay

$$\vec{s}_{\Box} = 0$$

Note helicities of neutrinos

$P$  

$C$  

$\Box^+$ at rest
Conclusions

1. Parity is not a good symmetry in the weak interaction. \((P)\)

2. Charge conjugation is not a good symmetry in the weak interaction. \((C)\)

3. The product operation is a good symmetry in the weak interaction. \((CP)\) - except in the kaon system!

4. Time symmetry is a good symmetry in the weak interaction. \((T)\)

5. The triple product operation is also a good symmetry in the weak interaction. \((CPT)\)